The Box Compression for Copy Paper Boxes – Applying McKee's Formula

Roman Popil, Ph.D.
Senior Research Scientist
Georgia Tech/Institute of Paper Science and Technology
Atlanta, Georgia USA

roman@gatech.edu

Abstract

A set of copy paper boxes were examined for the vertical compression strength (BCT), board caliper and edge compression strength (ECT) in the orientation of the vertical loading. A simplified version of the McKee formula was fitted to the data set and the resulting equation improves the prediction from 52% to 25 % average error. A further simplification in testing requirements is shown to be provided by replacement of ECT with an equivalent CD tensile stiffness measurement consisting of the product of the board basis weight and CD sonic velocity squared.

Introduction

Stacks of copy paper boxes are a ubiquitous fixture in most institutions corporate offices and office supply stores. As such, the stacking strength is usually not a concern since the product, packaged reams of paper consisting of 500 sheets to pack, provide the vertical support. The corrugated board serves to wrap the stack of paper reams and provide a surface for some colorful graphics for marketing purposes. People who elect to collect office copy paper boxes for moving purposes will discover to their dismay that stacked boxes carrying any load will collapse. Occasionally stacks of filled copy paper boxes stacked off the pallet by a photocopier or in an office corner will progressively lean over time and cause a vertical stack to topple casing some inconvenience.

The majority of copy paper boxes are constructed of B flute board but oddly, many have the side panels of the bottom tray containing the stack of paper reams oriented with MD in the vertical direction. This causes the boards to fail largely by buckling rather than by combined compression buckling as would occur in the case of a typical RSC shipping box.

The study reported here was initiated by a request of an industry Testing Services client to have a readily available means of discriminating between various boxes for potential stacking strength BCT. The interest is to determine which suppliers of copy paper to distributors packaged the manufacturer's products in the more reliable box without having to test for BCT.

McKee's formula for BCT

R.C. McKee and co-authors from IPC Appleton in 1963 [1] published a formula that has withstood the test of time as the simplest and most accurate predictor for box compression strength and is thus incorporated today in commercial box design software with various empirically based correction factors that provide some improved measures of accuracy. It is the author's own personal experience that the form of the McKee equation provides a good prediction of BCT for various laboratory made boxes consisting of 1) single wall A, B, C and E flute, 2) double wall EC, BC and AC boards, 3) F and N single wall 3) 30 point folding carton board. In each set of cases, the best agreement with experimental BCT values was obtained when the constants in the McKee formula were changed to fit the data.

McKee et al. started with the empirical observation and formula for the failure load of a rectangular plate vertically loaded. The edges of the plate are unrestrained and so are said to be simply supported using engineering mechanics terms. A box is really 4 connected corners with flaps and other features, but the failure load of simply supported plate is the starting point. The failure load of a plate P_p is the result of the combination of the compression strength P_s and the buckling strength P_{cr} in the following way:

$$P_p = C P_s^b P_{cr}^{(1-b)}$$

C is a constant and the exponent *b* is usually ¾. For corrugated boxes P_s is taken as *ECT*, the edge compression strength of the corrugated board. McKee et al. used the results derived for buckling load of a rectangular panel described in in a 1954 FPL report by Marsh. The form for P_{cr} is rather complicated so the McKee 1963 made several approximation that were considered not to affect the accuracy. The compression strength of a box BCT was taken as the compression strength of 4 panels P_p sufficiently high so that they will buckle when vertically loaded, and a square footprint box is assumed. This allowed a simple relationship between the *BCT* and the box perimeter *Z.* The buckling load for a panel is approximated by the form:

$$P_{cr} = \frac{C\sqrt{D_{MD} D_{CD}}}{w^2}$$

C is another constant, w the width of the panels *Z = 4w* and the *D* are the MD and CD bending stiffness of the corrugated board. Making the substitutions for P_p, P_s, and P_{cr} and combining the different constants into one, the McKee BCT becomes:

$$BCT = C\, ECT^{3/4} \left(\sqrt{D_{MD} D_{CD}}\right)^{1/4} \sqrt{Z}$$

If the side panels of the box are not high enough to allow panel buckling, the criterion from analysis being estimated as box height ≥ *Z*/7, then the box load fails entirely by compression only:

$$BCT = C \times ECT \times Z$$

To eliminate the need to measure the bending stiffness of corrugated board and to simplify calculations McKee made the approximation for bending stiffness *D* in P_{cr} as being equal to a sandwich beam of width *h* with tensile stiffness *E* x *t* of the liners of thickness *t* and modulus *E*:

$$D = \frac{Eth^2}{2}$$

Substituting the sandwich beam approximation for the D's produces the dependence of BCT on the square root of the board caliper *h*. The MD and CD tensile stiffnesses generally in a ratio of 1.5 for typical linerboard and are proportional to the ECT of the board. The proportionality is rationalized from the view that ECT can be modeled as a length weight summation of the compression strengths *SCT* of the board components liner and medium e.g., for single wall board:

$$ECT = C(2 \times SCT_{liner} + 1.42 \times SCT_{medium})$$

The compression strength in turn is proportional to the modulus E of the liner. McKee 1963 presented some representative data to illustrate the proportionality between tensile stiffness and ECT. Therefore,

$$ECT = CEt = CDh^2$$

These substitutions and approximations lead to the simplified form:

$$BCT = C\,ECT^{3/4}\left(\sqrt{D_{MD}D_{CD}}\right)^{1/4}\sqrt{Z}$$

$$= C\,ECT^{3/4}\left[\sqrt{E_{MD}th^2 E_{CD}th^2}\right]^{1/4}\sqrt{Z}$$

$$= C\,ECT\sqrt{tZ}$$

It is understood that the constant C changes with each successive substitution and approximation. To generalize from previous experience it is expected that the BCT of the copy paper boxes under investigation in this study will take the form:

$$BCT = C' \; ECT^a t^b$$

Since *Z* is constant for the current set of boxes it is incorporated in the proportionality constant *C'*.

Experimental

A set of 42 copy paper boxes post use were supplied by the client. Each box was either from a different manufacturer or design and were treated as distinctly different samples. 23 of the boxes were constructed such that the panels supported the vertical stack load in the CD of the corrugated board. Since copy paper boxes are loaded with paper ream stacks being higher than the panel height shown in Fig 1., the vertical stacking load is assumed by the paper ream stack

Figure 1. One of the copy paper boxes in the study loaded with paper reams and top tray removed.

therefore it is not of much immediate consequence whether the load orientation is in the CD or the MD of the board.

However, in the MD, the fluted medium does not contribute to the board compression strength and so ECT is comparatively less in the MD than in the CD about a factor of 2. B flute board when loaded in the MD in the T 839 clamp fails by buckling compared to failing by compression when the load is in the CD, the comparison is shown in Figure 2.

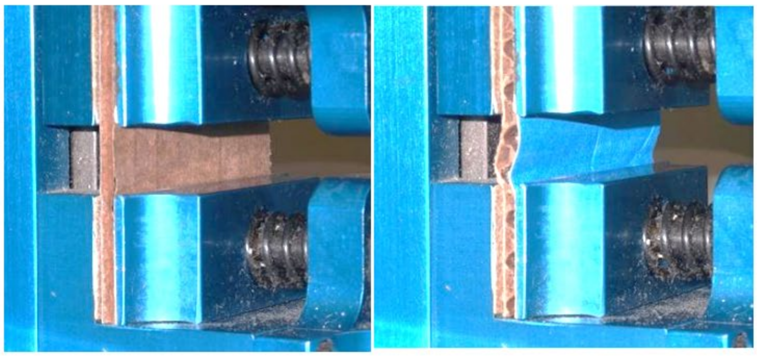

Figure 2. Close up side view of a copy paper B flute board at peak load during T 839 ECT with vertical load along the CD of the board (left) and load direction along the MD (right).

Sections of the bottom panels were cut and tested for board caliper and ECT using the T 839 clamp method. The ECT was measured with the load direction in either the CD or the MD of the board according to the box design.

The boxes and top trays were reassembled and tested for BCT using an Emerson 1100 compression tester, platen speed ½ inch per minute.

Fitting a McKee formula to the data

The data for board caliper, ECT and BCT were tabulated in an Excel spreadsheet. The form of the McKee equation was assumed not to depart far from the original:

$$BCT = C\ ECT^a t^b \sqrt{\{41t\}}$$

with *BCT* in lbs, *ECT* in lb/in, and *t* in inches. The perimeter Z of the boxes is 41. Arbitrary values for the constant and exponents were initially assumed and the absolute value of the difference from the model and actual BCT value calculated for each copy box data point. The Excel Solver function was used to minimize the sum of the differences by iteratively changing the constant *C* and exponents *a, b* obtaining the best fit:

$$BCT = 6.03\ ECT^{0.803} t^{0.844} \sqrt{\{41t\}} = 38.6\ ECT^{0.803} t^{0.422}$$

The fitted form of the equation has an average error of 25% calculated as the difference between model and actual values divided by the actual value. By comparison, the original McKee form used for RSC containers:

$$BCT = 5.87\ ECT\ \sqrt{\{41t\}} = 37.6\ ECT\ t^{0.5}$$

overestimates the BCT and has an average error of 52%, results shown in Figure 2.

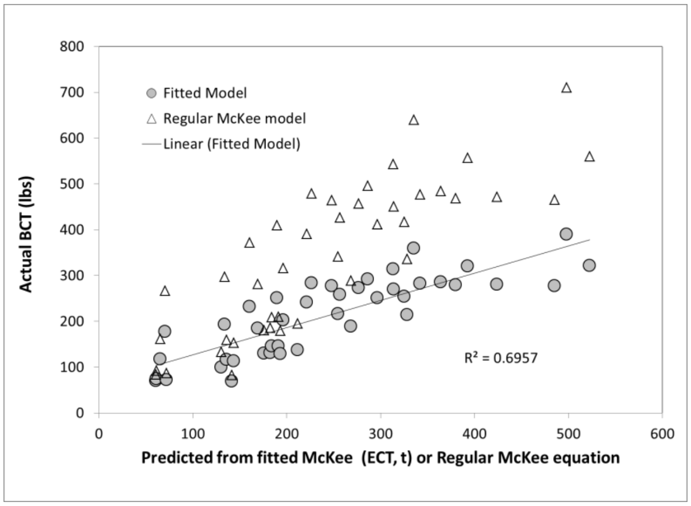

Figure 2. Comparison of actual BCT values with the fitted model McKee and the original McKee models for the copy paper box sub-set with the MD in the direction of load.

Replacing ECT with Ultrasonic Measurement and Basis Weight

Although the BCT of copy paper boxes can be reasonably estimated using the fitted McKee equation once values of ECT and caliper are known, it would be more convenient to eliminate the ECT since any ECT method requires the accurate cutting and/or preparation of the test sample. Since ECT is a test to failure, it is subject to any flaws in the test specimen such as non-parallel edges or any artifacts due to sustained damage from handling or manufacturing [5]. Therefore ECT is inherently a high variability measurement requiring a high average value to meet minimum spec requirements. In this section it is shown that using a combined measurement of sonic velocity along the board multiplied by the basis weight will lead to a value that is proportional to ECT and can be used as a substitute. The reasoning for this is as follows. The relationship between the speed of sound V density ρ and he modulus for a material is to a good approximation [6]:

$$E \cong \rho V^2$$

2 E ≈ ρV For an orthotropic material such as paper, the modulus and sound speed are directional and the direction of interest for ECT is usually the CD, thus:

$$E \cong \rho V_{CD}^2$$

Multiplying both sides of the above equation by caliper t, the left hand side is equivalent to the tensile stiffness S_T, the right hand side becomes the basis weight BW (= ρ x t) times the CD velocity squared:

$$S_T = BW \times V_{CD}^2$$

If the proportionality between ECT and the board stiffness is accepted on principle and recognizing the V_{CD}^2 is also the value produce by the L&W TSO instrument dubbed as TSI_CD, we have:

$$ECT \propto BW \times TSI_{CD}$$

The frequency used in the L&W TSO is 100 kHz and typical sound speeds in the CD along corrugated board are around 2.7 km/s. This means that the wavelength of the sound is 2.7 x 10^5 cm/s /(100 x 10^3/s) = 2.7 cm which is greater than the thickness of most common corrugated boards. Therefore, the sound propagates as longitudinal waves through the whole thickness of the board. The basis weight for corrugated boards can be easily measured to 0.5% or better variation using large pieces of about a square foot in area weighed on a balance accurate to a milligram. The variation in TSI is typically about 3% so the equivalent variation of is also about 3% which is half the variation of standard ECT measurements. Data for the entire copy paper box set is summarized in Table 1.

Table 1. Physical data summary for the 42 copy paper box sample set. TSI is the specific stiffness in the relevant direction of loading measured using the L&W TSO sonic tester. TSI x BW is the equivalent board specific stiffness calculated as the product of the basis weight and specific stiffness. "c.i." are 95% confidence intervals for corresponding average values. Last column, a "1" represents board direction with respect to vertical load.

Sample ID	caliper		ECT		BCT	basis wt	TSI		TSI x BW		CD is
	mils	c.i.	lb/in	c.i.	lb	g/m^2	(km/s)^2	c.i.	N/mm	error	vertical
HP Everyday (2)	109.3	2.9	23.9	1.55	133.7	451.1	3.92	0.04	1.77	0.024	1
Maple Leaf	111.4	2.6	29.65	0.9	160.6	511.6	4.31	0.14	2.20	0.075	1
Double A 3	89.8	6.3	25	2.35	169.1	562.3	3.84	0.07	2.16	0.045	1
HP Everyday (1)	104.3	3.9	26.05	1.35	196.1	442.5	4.26	0.06	1.89	0.033	1
Paperline 1	119.8	3.5	30	1.2	221.6	512.8	3.79	0.06	1.94	0.037	1
HP Multipurpose	110.9	1.2	38.3	0.95	226.3	514.1	4.67	0.08	2.40	0.047	1
Yes	122.2	1.5	35.3	1.35	248.1	568.3	4.07	0.03	2.31	0.028	1
Bola Dunia	112.8	1.6	33.75	1.55	256.6	613.6	3.83	0.05	2.35	0.040	1
WC Penfold	119.9	2.4	35.1	1.85	276.5	586.3	4.1	0.02	2.40	0.026	1
Sinarspectra	124.3	4.6	37.4	2.15	286.4	519	4.24	0.12	2.20	0.067	1
Multioffice	114	5.9	32.45	0.75	296.6	463.7	4.44	0.02	2.06	0.022	1
Galaxy Brite 1	115.6	2.6	42.5	1.95	313.7	607.7	3.93	0.15	2.39	0.094	1
Paperline 2	114.4	2.3	35.4	1.45	313.8	525.8	4.36	0.13	2.29	0.073	1
Smartcopy 1	115.3	4.1	32.7	2	325	569.1	4.11	0.03	2.34	0.028	1
Supreme	129.3	2.6	47.35	1.75	335.7	598.5	4.44	0.05	2.66	0.041	1
Smartcopy 2	114.7	5.2	37.45	1.6	342	571.6	4.09	0.04	2.34	0.031	1
Double A 5	116	3.5	37.8	2	364.3	546	4.36	0.02	2.38	0.026	1
Double A 4	115.3	2.1	43.6	2.25	392.8	580.6	4.37	0.07	2.54	0.048	1
Double A 2	117.3	9.6	36.6	1.05	423.8	573.2	3.97	0.06	2.28	0.042	1
Double A 1	115.8	0.9	36.35	1.95	485.1	562.1	3.92	0.02	2.20	0.024	1
Sinarline	123.4	4.3	53.8	2.05	498	726.8	7.64	0.11	5.55	0.100	1
Double A 6	111.6	3.9	44.55	3.45	522.7	595.8	4.26	0.07	2.54	0.049	1
Double A 7	115.8	7.3	36.65	2.6	379.7	540	4.35	0.02	2.35	0.025	1
Navigator 2	104.6	2.1	6.85	0.55	61	428.5	6.6	0.05	2.83	0.036	0
Rey	59	0	10.1	0.75	61.3	380.8	6.24	0.38	2.38	0.145	0
Future	62.6	2.9	17.15	1.65	65.4	370.1	6.97	0.07	2.58	0.037	0
Discovery	109.3	2.8	21.45	1.65	70.4	380.5	7.6	0.16	2.89	0.067	0
Navigator 1	110.6	1	6.95	1.2	72.1	434.5	6.38	0.15	2.77	0.070	0
Clairefontaine 3	58.1	0.4	14.65	0.85	130.2	417.8	6.48	0.20	2.71	0.088	0
Clairfontaine 2	62.8	2.4	16.95	3.45	136	488.9	7.23	0.16	3.53	0.085	0
Reflex	106.8	1.6	6.75	0.5	141.6	426	8.71	0.03	3.71	0.039	0
Staples	104.8	5.9	12.6	1.4	143.6	431	7.34	0.11	3.16	0.055	0
Paperone Copier Indo 1	110.1	2.1	14.5	2.65	175.7	579.2	7.09	0.03	4.11	0.044	0
Clairefontaine 1	64.7	2.8	19.45	3.1	182.4	519.9	6.92	0.39	3.60	0.204	0
PaperOne Copier China	116.5	6.4	16.25	2.1	183.9	575	6.18	0.09	3.55	0.062	0
Konica Minolta	120.7	2.7	31.35	2.35	189.6	581	7.19	0.09	4.18	0.066	0
Paperline Gold	117.5	2.3	16.25	3.25	191.4	659.4	5.19	0.04	3.42	0.045	0
Paperone Copier Indo 3	107	2.4	14.55	2.8	193.4	482.1	7.17	0.11	3.46	0.061	0
PaperOne All Purpose (Ch)	102.3	2.9	16.2	3.2	211.8	606.4	7.82	0.08	4.74	0.067	0
Smartlist	115.8	2.9	26.7	1.3	254.4	442.8	3.83	0.13	1.70	0.061	0
PaperOne All Purpose (In)	120.6	3.5	22.1	1.8	268.7	611.2	6.97	0.04	4.26	0.050	0
Paperone Copier Indo 2	121.7	0.5	25.6	2.55	328.3	653.6	7.68	0.03	5.02	0.053	0

Laboratory exploration of replacing ECT with (BW x TSI-CD)

An IPST study for factors affecting loaded corrugated box lifetime in cyclic humidity [7] had prepared a series of single walled A-flute test boxes which kept the liner facing basis weights constant at 42 msf but varied the fluted medium from 14 to 42 msf. The ECT of this board set was measured using the "neckdown" T 838 method since the clamping of method T 838 was found to adversely affect the lighter weight medium causing anomalously low ECT values to occur. Relevant physical values for this sample board set are shown in Table 2.

Table 2. Relevant data for an A flute sample set used to test the correlation of ECT to BW x TSI.

sample	medium bw gsm	board basis wt	ECT T 838 Neck Down (kN/m)	V^2 x BW
14A	68.3	511	5.9	2.48
16A	78.1	530	7.2	3.67
18A 723D	87.9	543	6.2	2.49
18 726E	87.9	540	7.8	3.76
20A	97.6	556	8.3	3.12
26A	126.9	603	8.6	4.16
33A	161.1	661	9.8	4.96
42A	205.0	733	9.1	4.73

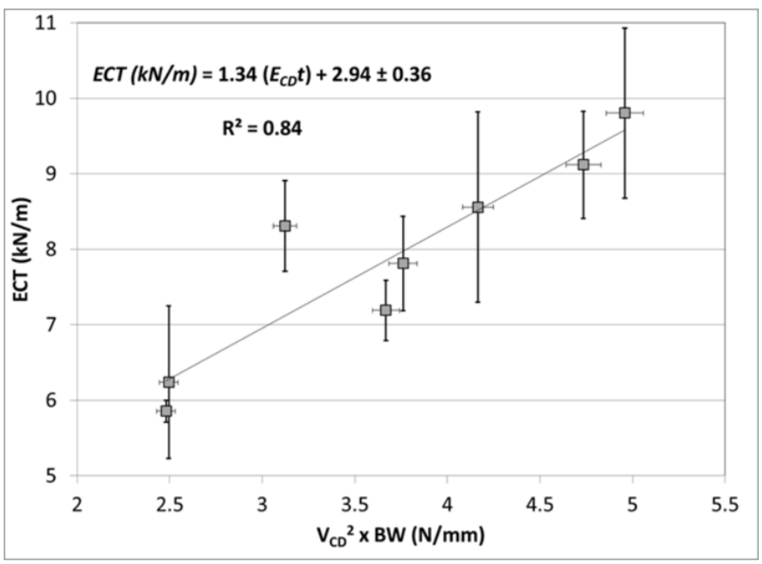

Figure 3. ECT of the A-flute sale set versus their ECT.

Figure 3 shows a fair correlation of the A –flute ECT values with (BW x TSI_CD), the equivalent board tensile stiffness. This shows that the range in strength of the medium is detected by the sonic propagation.

Application of (BW x TS-CD) to Copy Paper Boxes

The side panels of the copy paper boxes were measured for TSI $(km/s)^2$ also known as specific stiffness after the copy paper boxes were tested for ECT using areas that were free of creases. These values multiplied by the board basis weight produce the equivalent board tensile stiffness in units of kN/m or N/mm . When plotted versus ECT, the data subset with load in the CD of the board correlated well with ECT shown in Figure 4.

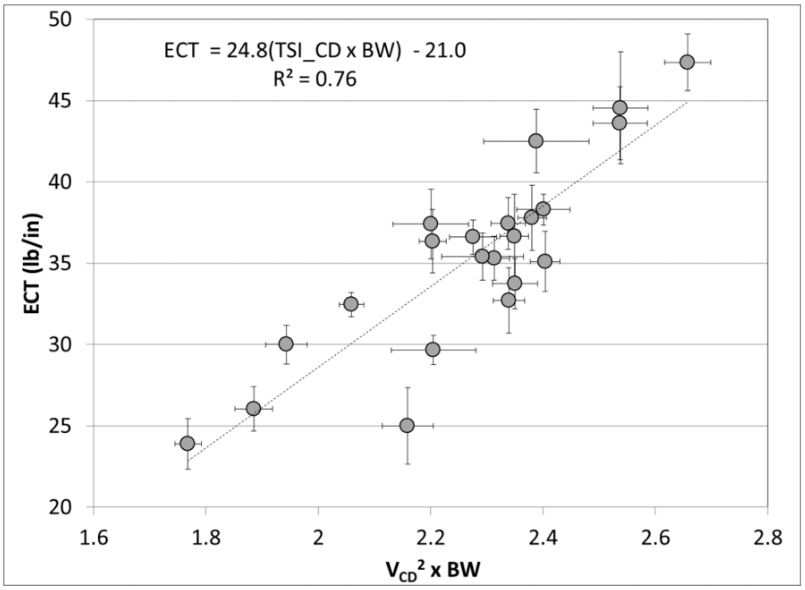

Figure 4. ECT versus TSI_CD (V_{CD}^2) x BW for the copy paper boxes for load applied in the CD of the board.

ECT versus TSI for the boards loaded in the MD also showed a similar but separate correlation with much more scatter. This is attributable to that when board is loaded in the MD, the medium does not contribute to ECT but still contributes to the measured sonic velocity in proportion to its basis weight.

Conclusions

By fitting the constant and exponents of an assumed simplified form of the McKee equation, the BCT of copy paper boxes can be reasonably predicted although the construction and loading orientation varies. The fitted form of the McKee can be used to assess which boxes will endure transport or storage conditions better if so required. ECT can be in principle be replaced by a combined measurement of board basis weight and TSI_CD. The potential here is the method can provide a quicker measurement without the need for cutting or other sample preparation and also has reduced variability in the results

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